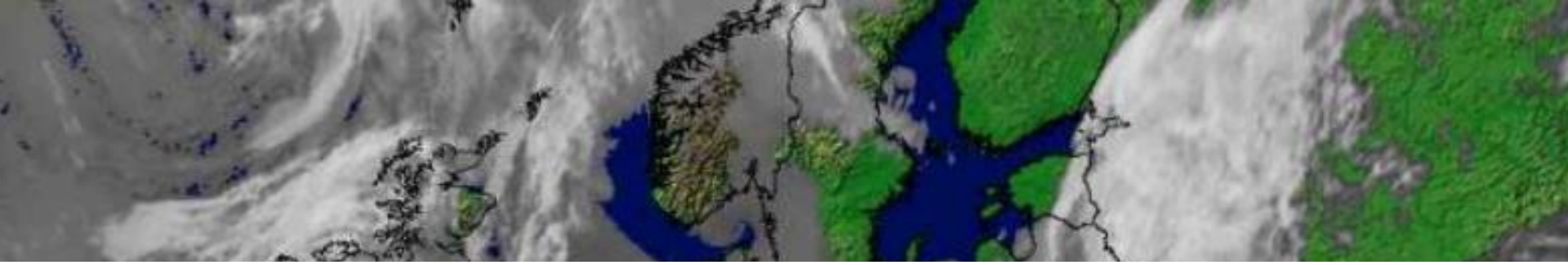




Application of Ensemble Transform Kalman Filter in Numerical Weather Prediction

Edit Adamcsek
adamcsek.e@met.hu



Numerical Weather Prediction



Numerical Weather Prediction

- Aim: give an estimation of the future state of the atmosphere, i.e. give a weather forecast.
- How? Numerically solve the **hydro-thermodynamic equations**, which describe the atmospheric flow.
- This is a system of non-linear Partial Differential Equations.

$$\frac{d\bar{v}}{dt} = -\frac{1}{\rho} \nabla p + \bar{g} - 2\bar{\Omega} \times \bar{v}$$

$$\frac{d\rho}{dt} = -\rho \cdot \text{div} \bar{v}$$

$$\frac{dQ}{dt} = c_p \frac{dT}{dt} - \alpha \frac{dp}{dt}$$

$$\frac{dq}{dt} = -\frac{1}{\rho} M$$

$$p = \rho RT$$

\bar{v} : wind

ρ : density

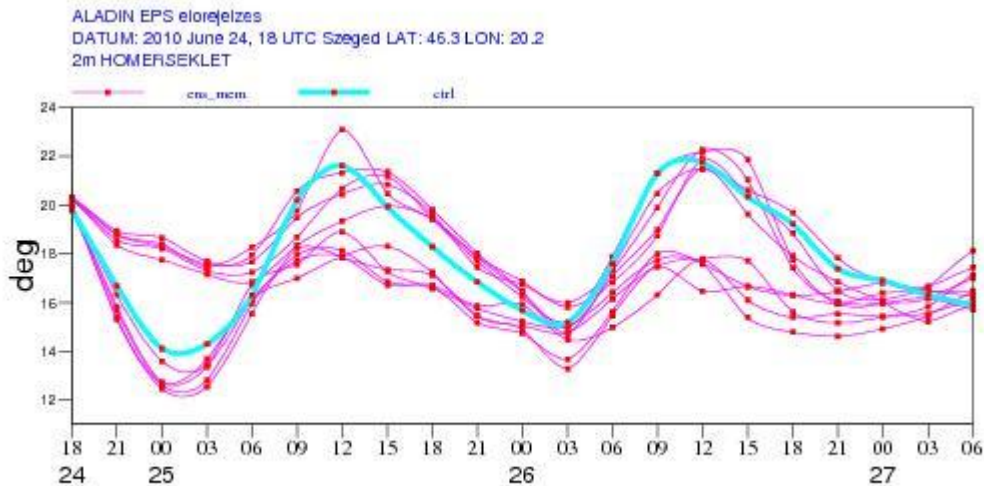
T : temperature

p : pressure

q : humidity

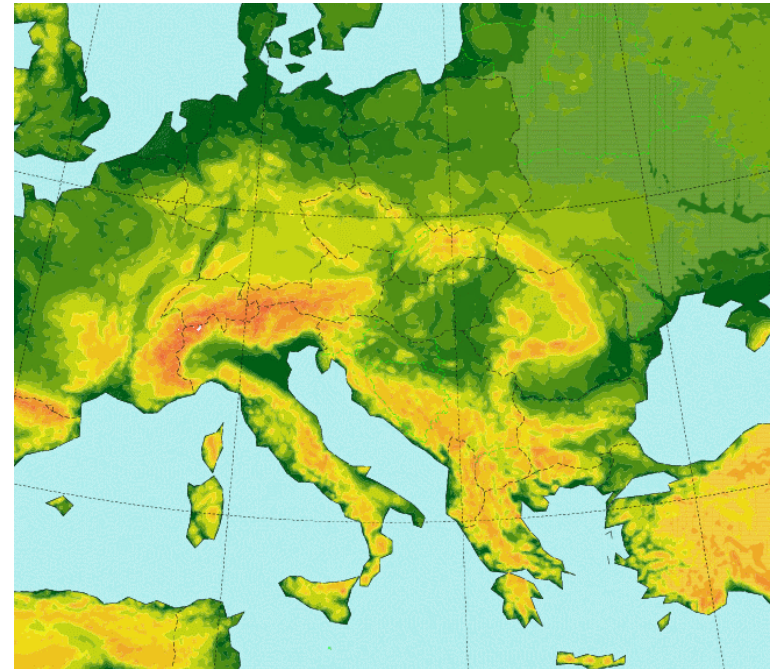
Data Assimilation

- The model needs **initial-** and lateral boundary **conditions**.
- The models are extremely sensitive to changes in the initial condition, so their precise estimation is crucial in NWP.
- **Data Assimilation** is the method to create the best possible estimate of the atmospheric state at the initial time, i.e. the analysis.



Operational ALADIN / HU

- ALADIN NWP model (cycle 33)
- LAM domain over Central Europe
- 8 km horizontal resolution, 49 vertical levels
- LBC from the IFS model (ECMWF)
- IC: local data assimilation
- analysis 6 hourly



Data Assimilation

- Analysis: the best possible estimate of the state vector.
- Two kind of information are used:
 - observations and
 - background information (a forecast from a previous analysis)
- The analysis is obtained by the minimization of the cost function:

$$E|x_t - x_a|^2 \rightarrow \min$$

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$$J(x) = \frac{1}{2} \underbrace{(x_f - x)P_f^{-1}(x_f - x)^T}_{\text{distance from the background weighted by the reliability of the forecast}} + \frac{1}{2} \underbrace{(y - H(x))R^{-1}(y - H(x))^T}_{\text{distance from the observation weighted by the reliability of the observation}}$$

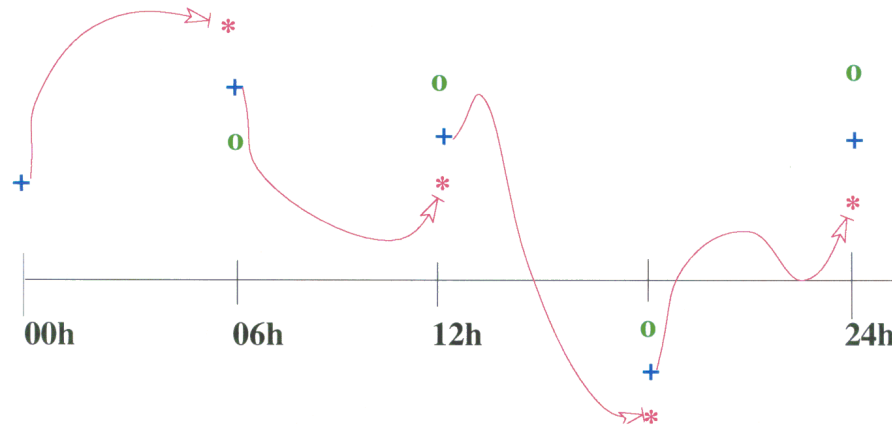
$$x_a = \arg \min J(x)$$

distance from the background
weighted by the reliability of the forecast

distance from the observation
weighted by the reliability of the observation

$x_t, x_a, x_f \in \mathbb{R}^n$ model space
 (true state, analysis, forecast)
 Observations: $y \in \mathbb{R}^p$
 Observation space $\subset \mathbb{R}^p$ ($p \approx 10^5$)
 Observation operator $H: \mathbb{R}^n \rightarrow \mathbb{R}^p$
 P_f : forecast error covariance matrix
 R : observation error covariance matrix

Data Assimilation



background + new observation \rightarrow analysis

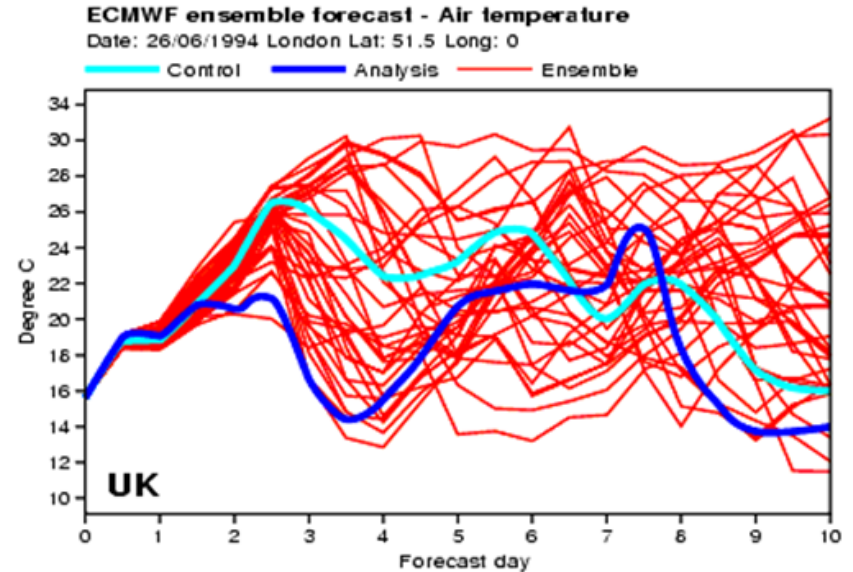
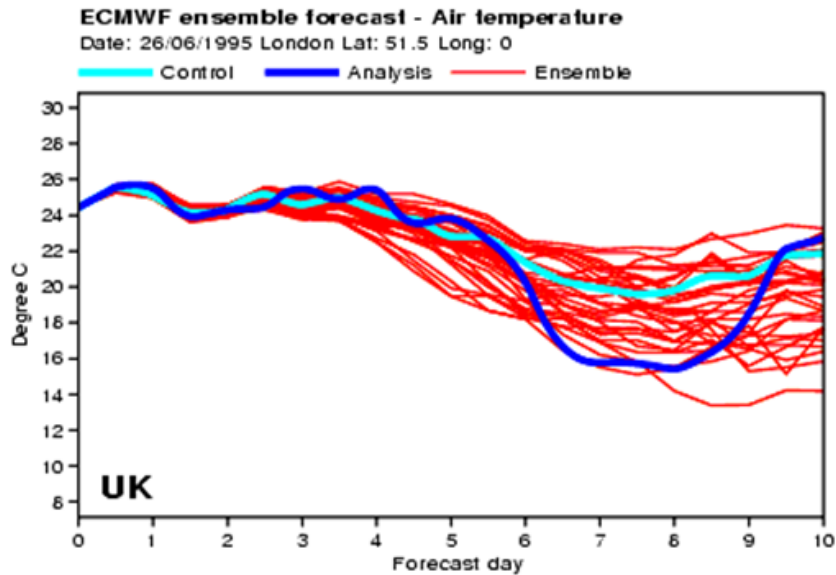
$$x_f(k+1) = Mx_a(k)$$

$$J(x) = \frac{1}{2}(x_f - x)P_f^{-1}(x_f - x)^T + \frac{1}{2}(y - H(x))R^{-1}(y - H(x))^T$$

$$x_a = \arg \min J(x)$$

Forecast error statistics are time-invariant in the model.

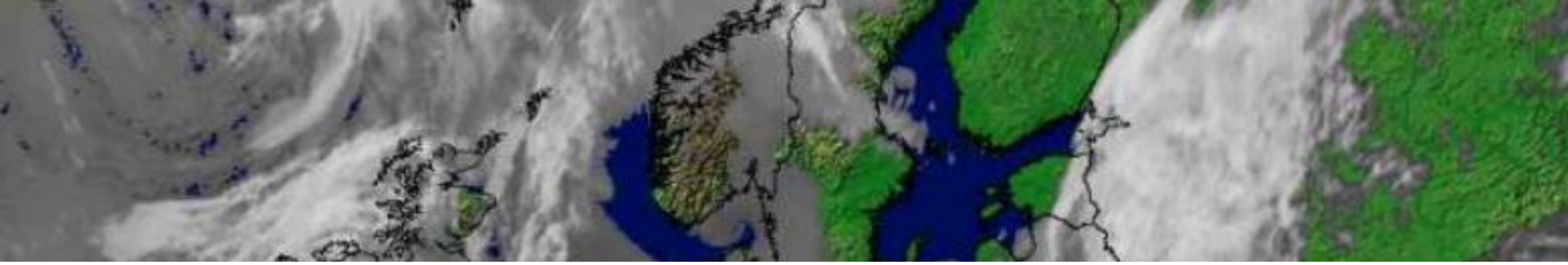
Forecast error



Ensemble forecast
London

1 year difference: 26/06/1995 and 26/06/1994

Aim of the research: flow-dependent computation of the forecast error covariance matrix.



Ensemble Transform Kalman Filter



Kalman Filter

Time evolution of forecast error:

$$P_f(k+1) = MP_a(k)M^T + Q$$

M linear model

P_a analysis error

Q model error

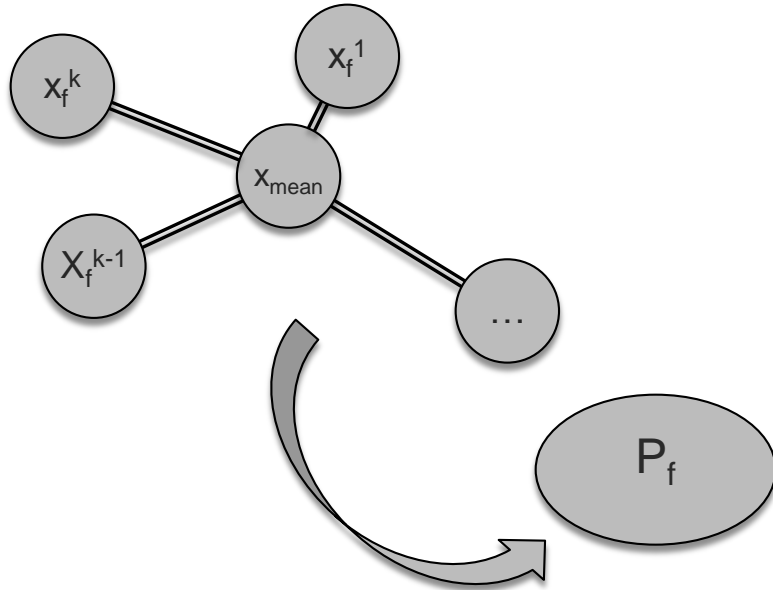


Kálmán Rudolf Emil
(1930-)

Ensemble Transform KF

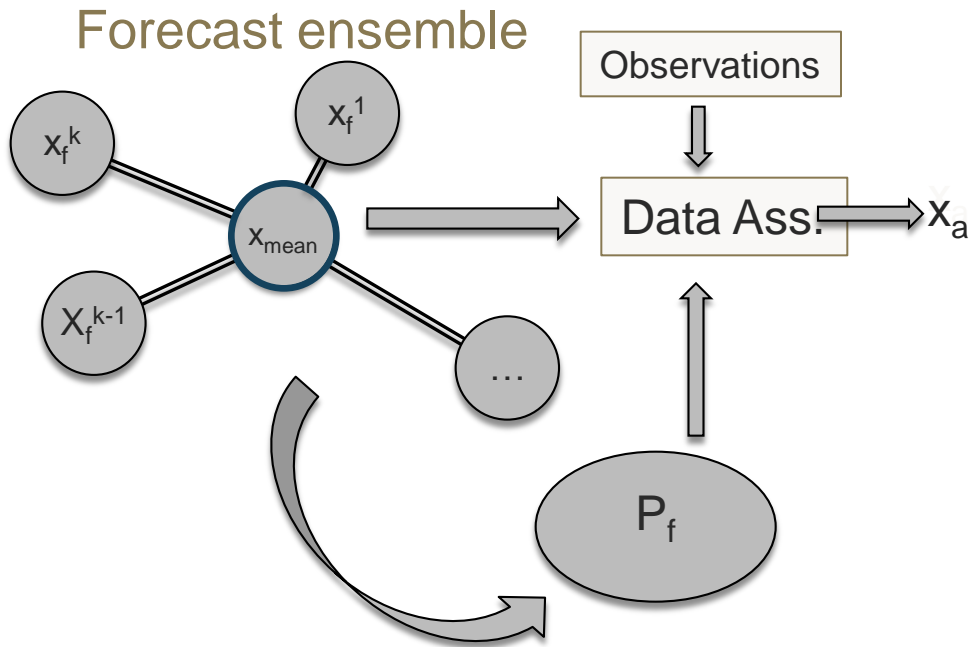
The forecast error (P_f) is represented by the forecast ensemble.

Forecast ensemble

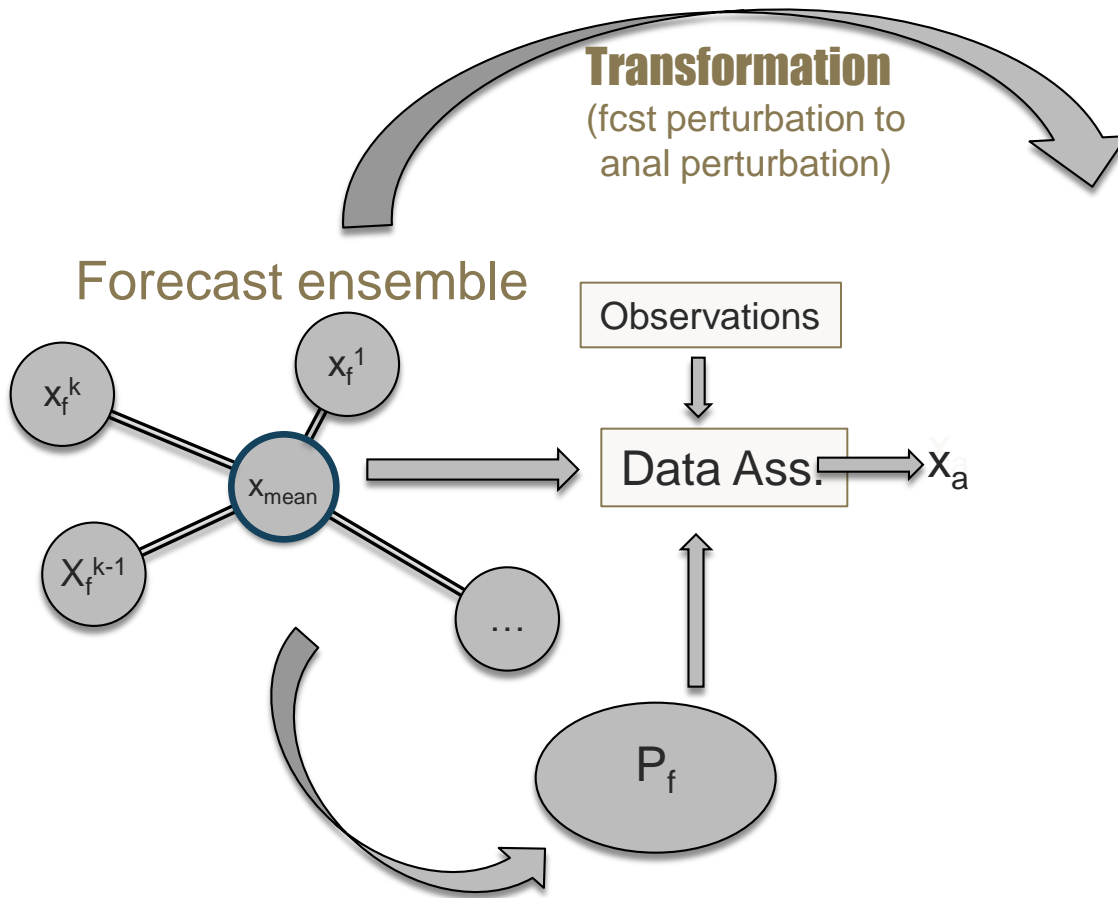


Ensemble Transform KF

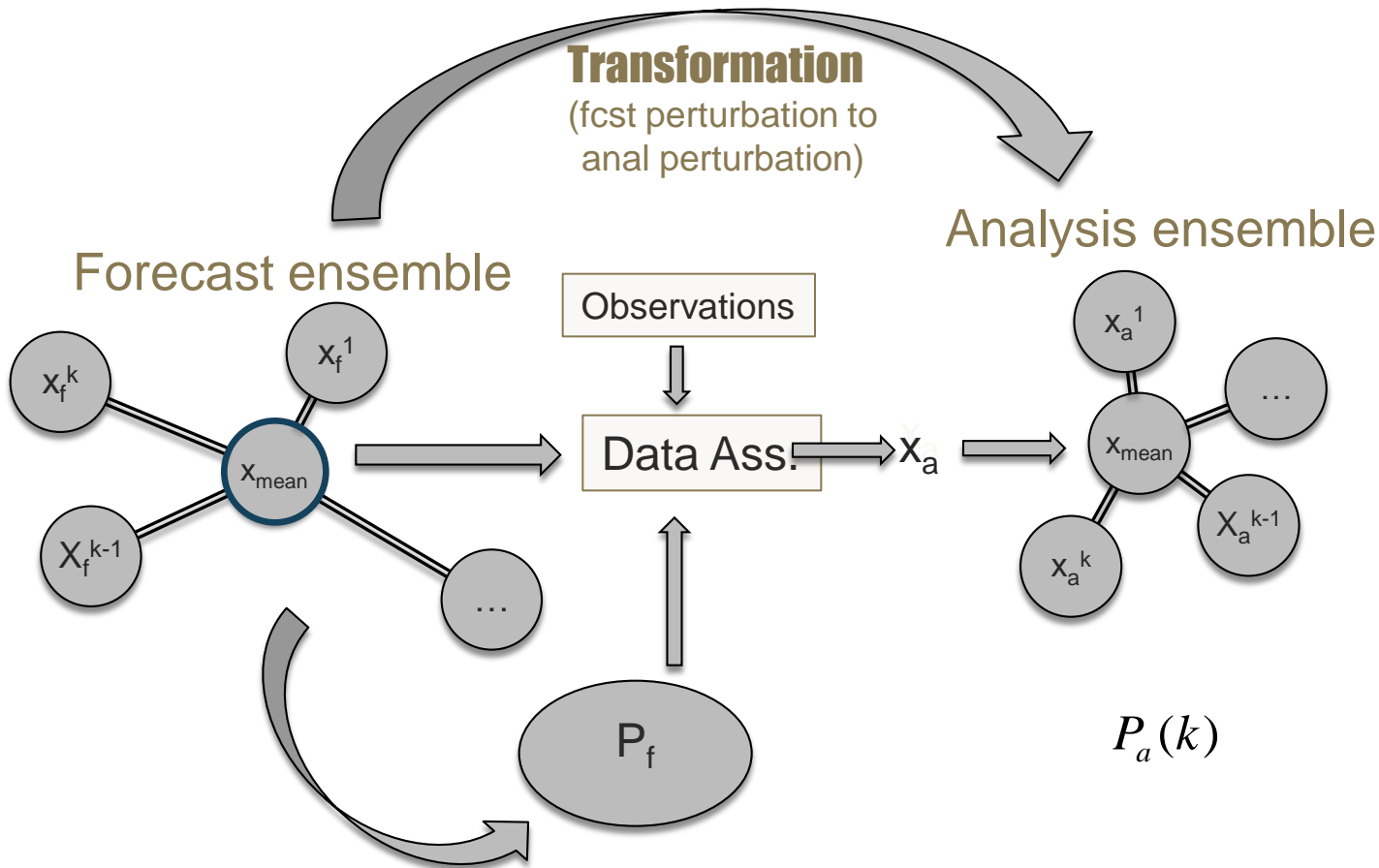
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Ensemble Transform KF

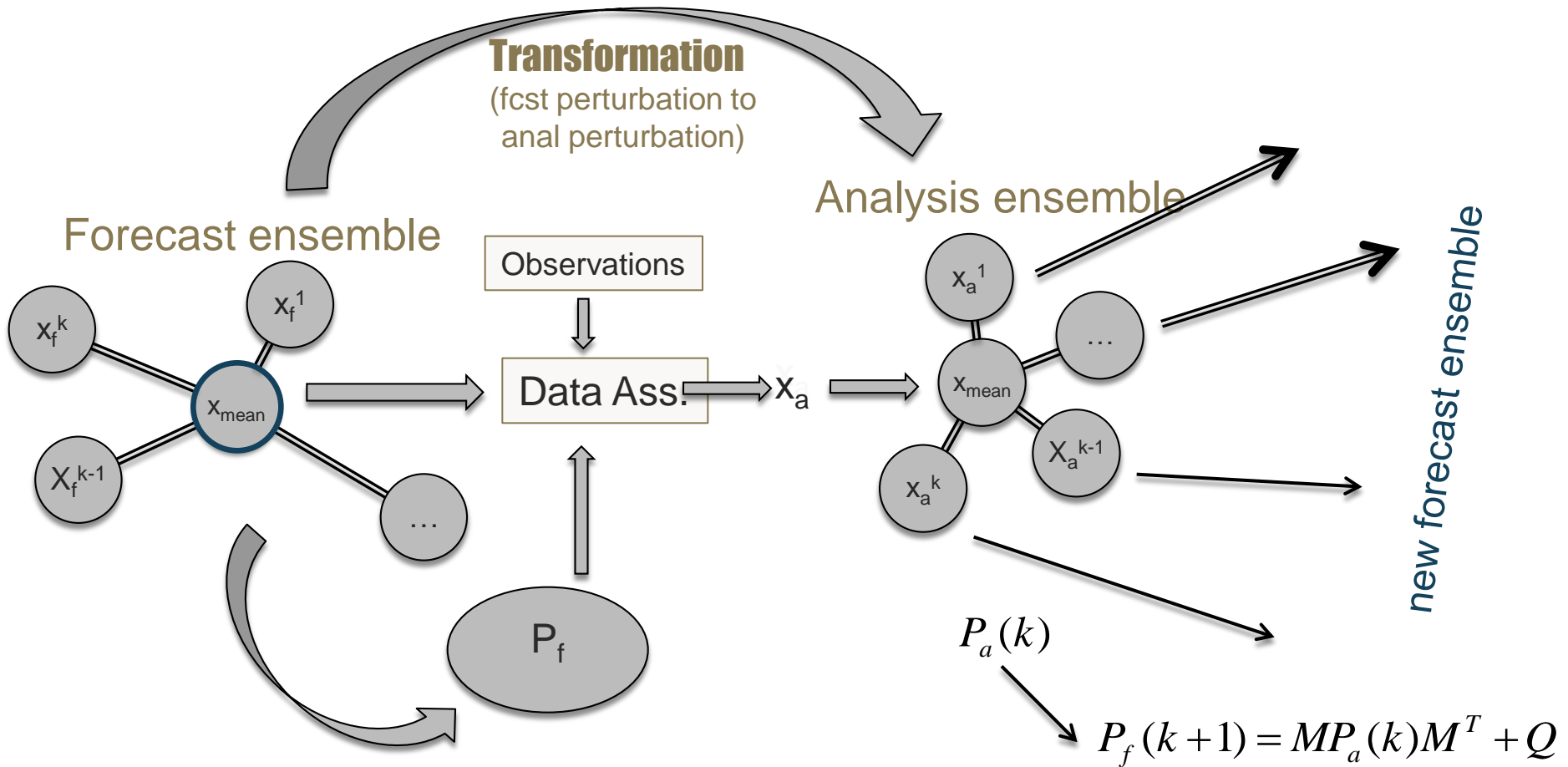


Ensemble Transform KF



The analysis error will be represented by the analysis ensemble.

Ensemble Transform KF



The new P_f will be represented by the new forecast ensemble.