Friday Afternoon Cloud Physics Talk (FACT), 27. April 2018.

Part II: Solving problems related to cloud physics Problems list

Problem 1. R. R. Rogers & M. K. Yau: A Short Course in CLOUD PHYSICS Problem 6.7. An air sample contains aerosol particles of sodium chloride with sizes ranging from D_0 to D_{max} and distributed according to the Junge form of problem 6.4. Nucleus counts are taken in a thermal-gradient diffusion chamber by observing the number of droplets that are activated as a function of the supersaturation. If $D_{max} \gg D_0$, show that the number of activated droplets is related to supersaturation by $N \propto s^2$.

The Junge distribution from 0.1 μm to 10 μm in diameter:

$$n_l(D) = \begin{cases} c \cdot D^{-3}, & 0.1\mu\text{m} \le D \le 10\mu\text{m} \\ 0, & \text{otherwise} \end{cases}$$

Problem 2. R. R. Rogers & M. K. Yau: A Short Course in CLOUD PHYSICS Problem 7.1. Probe that, in general, the difference between the masses of two droplets growing in the same environment according to the approximation (1) increases with time.

$$r(t) = \sqrt{r_0^2 + 2 \cdot \xi \cdot t} \qquad (1)$$

where $\xi = \frac{(S-1)}{[F_k + F_d]}$, where F_k represents the thermodynamic term that is associated with heat conduction; F_d is the term associated with vapor diffusion.

Problem 3. R. R. Rogers & M. K. Yau: A Short Course in CLOUD PHYSICS Problem 7.4. A sample of moist air is cooled isobarically. A cloud forms and the cooling continues. In a form similar to (2), the rate of change of the saturation ratio may be written:

$$\frac{dS}{dt} = q_1 \cdot \frac{dT}{dt} - q_2 \cdot \frac{d\chi}{dt}$$

where dT/dt and $d\chi/dt$ are the rates of change of temperature and condensed water. Derive expressions for the thermodynamic factors q_1 and q_2. Evaluate these expressions for $p = 80\ kPa$ and $T = 280\ K$.

Where (2) is the following:

$$\frac{dS}{dt} = Q_1 \cdot \frac{dT}{dt} - Q_2 \cdot \frac{d\chi}{dt}$$

where $Q_1 = \frac{1}{T} \cdot \left[\frac{\varepsilon \cdot L \cdot g}{R' \cdot c_p \cdot T} - \frac{g}{R'}\right]$ and $Q_2 = \rho \cdot \left[\frac{R' \cdot T}{\varepsilon \cdot e_s} + \frac{\varepsilon \cdot L^2}{p \cdot c_p \cdot T}\right]$.

Problem 4. R. R. Rogers & M. K. Yau: A Short Course in CLOUD PHYSICS Problem 7.5. To analyze the approximate behavior of the supersaturation in a cloud of growing droplets, suppose the droplets are all the same size, growing by condensation according to (1). Let ν_0 denote their concentration per unit mass of air, and regard this quantity as a constant under the assumption that no new drops are created and that no existing drops are lost. Show that these assumptions, taken in connection with (2) lead to $ds/dt = \omega - \eta \cdot s$, where $\omega = 100 \cdot Q_1 \cdot U$ and $\eta = 4 \cdot \rho_L \cdot \nu_0 \cdot r \cdot Q_2/(F_k + F_d)$. (Denote, that U is the vertical velocity: dz/dt). Assuming further that the temperature, pressure and droplet size are slowly varying compared to the supersaturation *s*, find the solution of this equation that satisfies the initial condition $s = s_0$ at $t = t_0$, Show that the supersaturation tends to $s_\infty = \omega/\eta$ as the time increases, and that the relaxation time of the supersaturation is η^{-1}.

Problem 5. R. R. Rogers & M. K. Yau: A Short Course in CLOUD PHYSICS Problem 8.4. A drop of 0.2 mm diameter is inserted in the base of a cumulus cloud that has a uniform liquid water content of 1.5 g/m^3 and a constant updraft of 4 m/s. Using the elementary form of the continuous-growth equation and neglecting growth by condensation, determine the following:

(a) the size of the drop at the top of its trajectory;
(b) the size of the drop as it leaves the cloud;
(c) the time of the drop resides in the cloud.

Assume a collection efficiency of unity, and for the dependence of fall velocity on size use the data in Table 1. (Note: Parts (a) and (b) of this problem are well suited for graphical solution.

The continuous-growth equation is the following:

$$\frac{dR}{dz} = \frac{E_m \cdot M}{4 \cdot \rho_L} - \frac{u(R)}{U - u(R)}$$

where E_m is the effective average value of collection efficiency for the droplet population and *M* is the cloud liquid water content in units of mass per unit volume, *U* is the vertical velocity and *u*(*R*) is the terminal fall speed of the drop.

Problem 6. R. R. Rogers & M. K. Yau: A Short Course in CLOUD PHYSICS Problem 8.6. A small drizzle drop is swept upwards in a cumulus congestus cloud and grows by accretion and condensation in the supersaturated environment. The condensation parameter ξ may be regarded as constant and the linear fall speed law of problem 8.1 approximated the relative velocity between the growing drop and the cloud droplets. Develop and solve the differential equation that describes the growth of the drop by accretion and condensation acting simultaneously. Compare the result with the approximation obtained by adding the solutions for growth by accretion and by condensation acting separately.

The linear fall speed law is the following: $v = k_3 \cdot r$, where $k_3 = 8 \times 10^3\ s^{-1}$.